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Developing Pupil Profiles In Primary Mathematics Using Item Response Theory

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Abstract

As part of a multi-disciplinary research[+] entitled **Primary Pupil Profiling Project**, a team of Mathematics educators had developed a set of test instruments which could provide profiles of pupils in their performance of basic skills and processes across the primary mathematics curriculum. These profiles also provide semi-diagnostic information for teachers to assess the weakness and strength of individual pupils in three areas, viz. Number, Measurement and Geometry. The tests were of paper-and-pencil multiple-choice format for Grades 3 and 5. In each topic a sequence of attainment levels was identified based on the actual difficulty of learning tasks and ability of the learners. The lower levels consist of elementary knowledge and skills while the higher levels reflect conceptual understanding and use of problem solving processes. Item response theory was applied to analyse data and to present results in the form of kidmaps that are meaningful to pupils, their parents, teachers and the school as a whole.

Introduction

The word *profiles* refers to formats for portrayal of students and is often used synonymously in a narrower sense with the term *record of achievement*. A profile according to Law (1984) is a panoramic representation: numerical, graphical or verbal, of how a student appears to assessors across a range of qualities or in respect to one

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quality as seen through a range of assessment methods. Many of the conventional formats offer very scant statements about students; usually the spaces for 'grades' and 'comments' do not encourage teachers to say much about how the grade was achieved or on what observation the comments are based. Such traditional formats offer little information to the students themselves about how and why they came to be assessed in that way. By developing a profile with more continuous and comprehensive information about students, teachers will be providing themselves with a useful resource on how effective is their teaching, how to help students and at the same time portray students to others such as parents. For example, Masters and Doig (1992) developed a profiling method which takes the form of an ***individual map*** to portray a picture of learning from the perspective of the learner which focuses on what the child actually does when he or she solves mathematical problems.

In mathematical assessment, educators are questioning the validity of aggregation of test scores as a record of achievement. Is it valid to total the number of items scored correctly on a test as an indication of a students' knowledge of mathematics? Stiggins (1988) supports the view that teachers consider assessment important and much of their time is engaged in assessment activities. Teachers may use a variety of assessment techniques but the paper-and-pencil test is the most dominant practice in the mathematics classroom. In Singapore, large scale paper-and-pencil assessment of mathematics achievement at the national level exerts a powerful influence on teaching and learning of mathematics. Teachers are mindful of the content that is tested and concerned about the quality of assessment. Most teachers adopt the conventional behavioural approach in assessment accompanied by definite content specifications which facilitate their writing of detailed paper-and-pencil test items.

However, conventional testing has its limitations. For example, when a pupil takes a test, we realise that the raw score obtained by the pupil depends on both the ability of the pupil as well as the difficulty level of the test. Thus, a pupil with a test score of 58 in one test may have the same ability level as another pupil who has a test score of 72 in another test of the same kind, if the first test is more difficult than the second test. This is to say that the meaning of the total test score is not defined unless more information is known about the items in the test. Rather than forming an arbitrary scale according to difficulty levels from a collection of facts, skills and concepts, a statistical approach based on the Rasch Item Response Theory (Kwan & Shannon, 1989) can be used to select test items that fall along a linear scale according to difficulty and that individuals can be sequenced on the same scale by ability.

To achieve such objectivity in testing (Willmott & Fowles, 1974), at least two conditions must be met, namely: (a) estimates of attainment of a pupil are independent of the particular set of items which comprise the test, and (b) the characteristic of the test items such as ease or difficulty is also independent of the distribution of attainment of the pupils who are given the test. The test items, if they have been properly constructed, can be ranked from easy to difficult from empirical data, and so they define a scale of attainment corresponding to their levels of difficulty. To measure a pupil's level of attainment is equivalent to estimating an appropriate location on this scale, and this can be done by making use of his or her responses to the test items.

When a pupil attempts to answer a test item, a correct response depends as much on his/her ability as on the item difficulty. Therefore we can expect a pupil's chance for success to increase with ability and to decrease with item difficulty. This is central to the item response theory which allows for the estimation of a person's ability on the same latent continuum scale, independent from the subset of items that have been designed to fit the model. The basic unit of measurement of the item response models is the item. This is to say that, if a pupil's ability is higher than the level of difficulty of a item, we can expect a correct response; on the other hand, if the pupil's ability is lower than the level of difficulty of the item, then we can expect an incorrect response.

Rasch approaches have been used in development of mathematical operations test (Cornish & Wines, 1977) and test for measuring problem-solving ability of nonroutine mathematical problems (Malone, Douglas, Kissane & Mortlock, 1980). In the same way, this study makes use of item response theory to construct a cognitive hierarchy of attainment levels for profiling pupils in the learning of primary mathematics. These profiles also provide semi-diagnostic information for teachers to assess the weakness and strength of individual pupils in three areas, viz. Number, Measurement and Geometry at Grade 3 and Grade 5. Portrayals of pupils' performance are presented in the form of kidmaps that are meaningful to pupils, their parents, teachers and the school as a whole. The development of the Number, Measurement, and Geometry tests in this research has been strongly influenced by the success of the Basic Skills Testing Program (BSTP) in New South Wales, Australia (Masters et al., 1990). The BSTP Program used item response theory techniques to map student achievement with respect to a set of defined skill levels and to provide feedback to parents and teachers in a form that will be useful in helping students to build upon their current achievement.

Purpose of the Study

As part of the multi-disciplinary research project on Primary Pupil Profiling involving English, Mathematics, Chinese, Learning Abilities and Disposition, the Mathematics component which is reported here, has two main objectives:

1. to develop and validate assessment tools on Number, Measurement and Geometry for use in Grade 3 and Grade 5 school-based assessment;
2. to profile mathematics performances of pupils in the form of kidmaps that are meaningful to pupils, their parents, teachers and the school as a whole.

Stages in the Development of the Tests

Six test instruments were developed in this study: three tests were for Grade 3 and another three for Grade 5, covering three main topics: Number, Measurement, and Geometry. The project started in August 1991 and two trials were involved in the construction of the test items to ensure that the instruments developed had content and construct validity. The scope and objectives of the relevant sections of the primary level mathematics determined the content validity of the tests. School teachers were consulted in selecting and constructing items which would be relevant to the contents and processes that pupils were being exposed to. This was also to ensure that various aspects of the curriculum were appropriately represented.

At the start, for each test an item bank of 30 to 40 multiple-choice items assigned to their respective prior levels, were collected. A workshop was then arranged for teachers who were teaching Grades 3 or 5 in two schools, to evaluate and improve on the items. At an assigned date in August 1992, the six sets of tests were given to 151 third grade (G3) pupils and 139 fifth grade (G5) pupils of the two schools. Each pupil took the three tests (Number, Geometry and Measurement in this order) at one sitting. Whenever pupils were through with a test, they were told to raise their hands so that the teacher would assign them the next test. Generally, the majority of the pupils were able to complete a test within 30 minutes. The test data collected from the two schools were subjected to a computerised analysis using a Rasch analysis program called Quest (Adams & Khoo, 1991).

Item Calibration and Attainment Levels

The van Hiele theory of levels of geometric thought and the Bloom's taxonomy of levels of cognitive domain for number and measurement were used initially as guidelines in determining a prior basis for assigning tasks to the various attainment levels. The study found that different emphases of teaching and curriculum could alter the difficulties of some of the test items selected at the beginning. For example, some items which test knowledge and conceptual understanding designed for the lowest level, were found to be more difficult for pupils than those items on application of routine skills involving one-step or two-step word problems. As the emphasis by teachers in the classroom was on application of routine skills, the pupils' performance of these skills was found to be more advanced than their understanding of underlying concepts. With curricular change in the new syllabus which emphasises teaching for meaning on the development of concepts, this would eventually change the balance. For the purpose of this study which was based on empirical data obtained from two schools, four attainment levels were established for all the tests and were scaled as follows:

Level 1 $x < -1.0$ logits,
Level 2 $-1.0 \leq x < 0.0$ logits,
Level 3 $0.0 \leq x < +1.0$ logits,
Level 4 $x \geq +1.0$ logits.

This logit scaling was based on Rasch analysis of the data for the G3 and G5 samples. It shows the calibration of item difficulty and pupil ability for the various test items being plotted along the same vertical scale. Appendix B is an example for the G3 Geometry Test showing details of the levels and their item properties while Appendix C shows the common scales where these item difficulties are represented in terms of pupils' ability. Appendix A shows all items of the G3 Geometry Test. Full details of the six test instruments, their item analysis and pupil performance in each test, can be found in Foong, Yap, Khoo and Kaur (1994).

Results of Overall Pupil Performance

The overall performance of pupils in this pilot project is summarised in Tables 1 and 2. Table 1 shows the mean and standard deviation of scores of the G3 and G5 pupils for each of the three topics. The results are expressed in terms of the scores calculated on the various logit scales obtained from the calibration of the individual instruments. For example, the mean score of -0.72 on the G3 Geometry Test corresponds to the average value of the distribution of pupil ability estimates depicted in Appendix C. A G3 pupil who scored -0.72 on this test has at least 50% chance of correctly answering items with difficulty estimates of -0.72 or lower on the test but he has less than 50% chance of succeeding on items which are higher up on the scale.

For reporting purposes, the levels of attainment have been translated into skill bands. For example, in Figure 1, a child's levels of attainment in the three topics are translated into skill bands which describe the types of tasks that he would most likely be able to do. Table 2 shows the percentages of G3 and G5 pupils with results in the different skill bands. The information from this table complements that from Table 1 in providing an overall picture of the pupils' performance.

Table 1: Overall Performance of Pupils in the Pilot Project

Group	No. in group	Number		Measurement		Geometry	
		mean	s.d.	mean	s.d.	mean	s.d.
Grade 3	151	0.52	1.12	0.63	1.00	-0.72	0.74
Grade 5	139	1.05	1.12	1.02	1.05	-0.44	0.70

Table 2: Percentage of Pupils in the Four Skill Bands

Test	Group	Band 1	Band 2	Band 3	Band 4
Number	Grade 3	4.6	25.8	39.1	30.5
	Grade 5	2.9	15.8	20.9	60.4
Measurement	Grade 3	3.3	20.5	42.4	33.8
	Grade 5	4.3	5.8	36.0	54.0
Geometry	Grade 3	37.1	49.0	13.3	0.7
	Grade 5	21.6	56.8	18.0	3.6

For Number and Measurement, the majority of the G3 pupils were placed in Band 3 (39.1% for Number, 42.4% for Measurement) while the majority of the G5 pupils were able to attain the highest level at Band 4 (60.4% for Number, 54.0% for Measurement). This indicates that most of the G5 pupils were successful in using higher order skills in these two topics. As for Geometry, the majority of both grades were able only to attain the lower level at Band 2 (49.0% for G3, 56.8% for G5), indicating that pupils, even at G5, were less successful in analysing properties and relationships between geometric shapes.

Reporting Pupil Performance Profiles

Figure 1 shows the format of 'report card' or profile of pupil's performance that parents will receive, depicting in more graphical details the ability levels of the child across and within each area of the mathematics he or she has attained. These methods capitalise on item response theory to provide more informative reports to schools, teachers and parents than are possible with more traditional test analysis methods. This approach of reporting uses less numbers but more pictures and words to describe pupils' test results. This will help to make reports as informative as possible to teachers and parents and reduce the chance of inappropriate interpretations - such as interpreting a certain number as a 'pass mark'.

Central to these profiles are the attainment levels developed for each aspect of the test. The level of attainment of the pupils is presented in terms of the four skill bands which correspond to the four attainment levels developed for each aspect. Additional detailed feedback on the performance of individual pupil can also be provided to help parents and teachers design learning activities and experiences appropriate to the pupils' current levels of achievement.

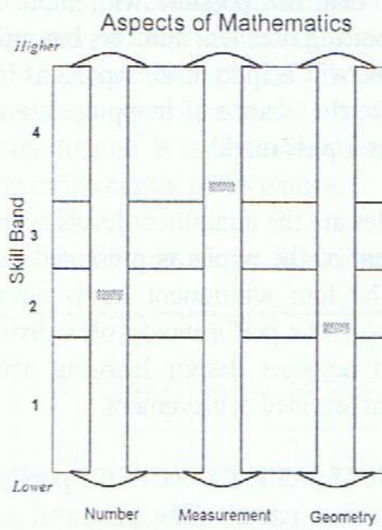
Figure 1 is an example of profile for one of the pupils tested in this study and is referred to here as 'Lin' (fictitious name). The estimated achievement levels in the three aspects of the tests are indicated by shaded rectangles. It can be easily seen at one glance that Lin performed at a higher level in Measurement than in Number; his performance is lowest in Geometry. His test performances place him near the boundaries of Bands 2 and 3 for Number, near the boundaries of Bands 3 and 4 for Measurement and near the boundaries of Bands 1 and 2 for Geometry.

The section of the report, headed 'Your Child's Skill Levels', interprets Lin's test results in terms of the kinds of knowledge and skill typical of pupils with his profile of achievements. In Number, for example, Lin's test performance places him near the boundaries of Bands 2 and 3. Since the shaded rectangle representing his numerical result is slightly below the boundary, his test performance has been assigned to Band 2 and his performance is described in terms of Band 2 skills. Pupils at this level of skill can typically solve two-step word problems involving '+' and '-', combine and split number relationships in a number statement involving any two operations and solve comparison word problems involving relationships of "more than", "less than" and "same as".

Figure 1. Sample of Report for Parents

REPORT FOR PARENTS

Pupil: Lin Grade: 3
School: XYZ School



Your Child's Skill Levels (shown above as ▒)
Pupils at this level are typically able to

Number - Band 2*

- solve two-step word problems involving '+' and '-'
- combine and split number relationships in a number statement involving any 2 operations
- solve comparison word problems involving relationships of "more than", "less than" and "same as"

Measurement - Band 4*

- solve "inverse" problems (e.g. find the side of a square given its perimeter)
- solve word problems on measurement which require logical deduction and/or inference

Geometry - Band 2*

- compare angles with a right angle or with each other
- identify shapes from given attributes
- identify attributes of a rectangle/square

*Your child's skills also include those described for skill bands up to this level

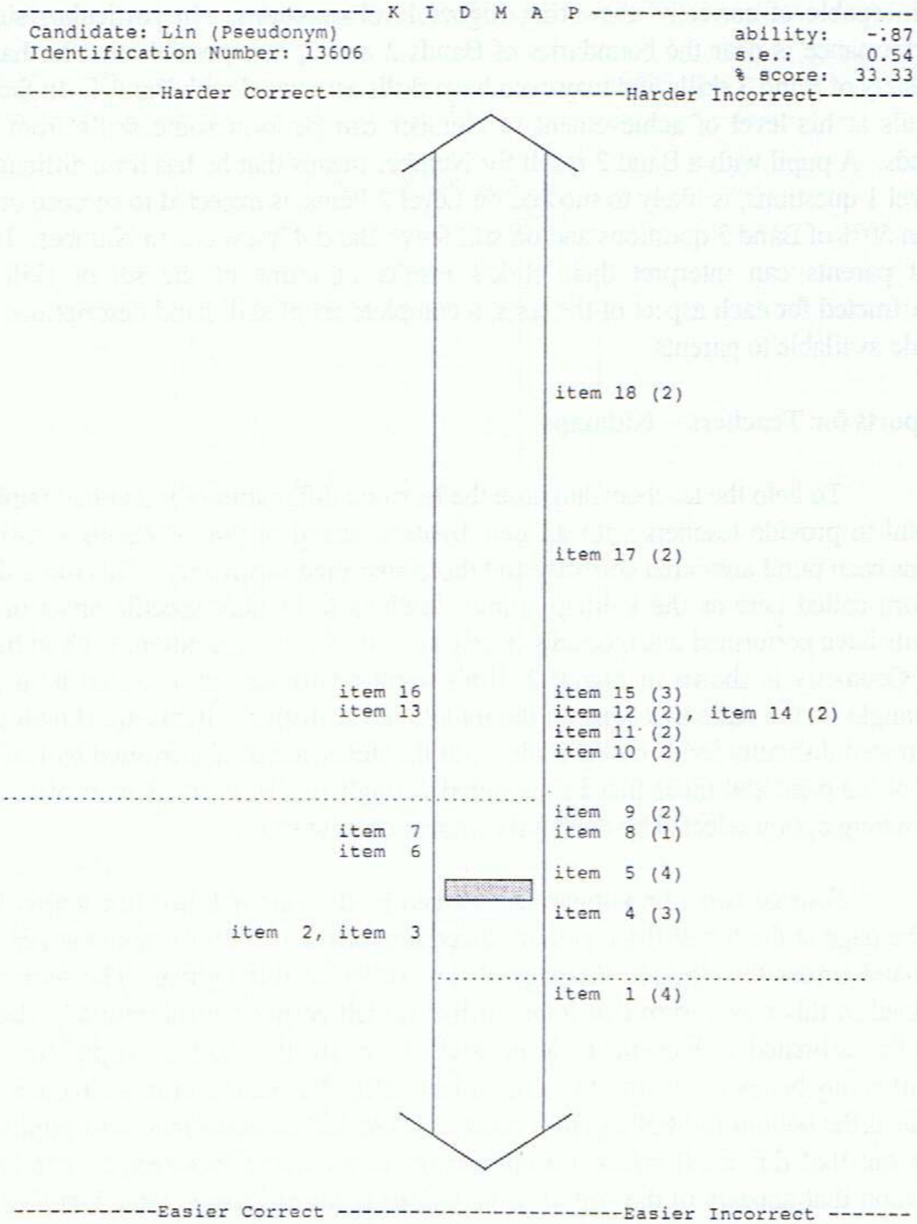
The fact that Lin's test result in Number places him in Band 2 does not mean that he will always answer Level 2 and Level 1 type of Number questions correctly and is incapable of correctly answering higher level questions. In particular, since his performance is near the boundaries of Bands 2 and 3, it is possible that he has some mastery of Band 3 skills and may even have skills associated with Band 4. In fact, most pupils at his level of achievement in Number can perform some skills from higher bands. A pupil with a Band 2 result for Number means that he has little difficulty with Level 1 questions, is likely to succeed on Level 2 items, is expected to succeed on fewer than 50% of Band 3 questions and on still fewer Band 4 questions in Number. In order that parents can interpret their child's results in terms of the set of skill bands constructed for each aspect of the tests, a complete set of skill band descriptions is also made available to parents.

Reports for Teachers – Kidmaps

To help the teachers diagnose the learning difficulties of individual pupils, it is useful to provide teachers with an item-by-item record of pupils' results showing the items each pupil answered correctly and those answered incorrectly. This more detailed report, called here as the kidmap, allows teachers to identify specific areas in which pupils have performed unexpectedly poorly or well. As an illustration, Lin's kidmap for G3 Geometry is shown in Figure 2. Lin's result on the test is indicated by a shaded rectangle on the scale that runs up the middle of the display. Items are shown at their estimated difficulty levels on the scale, with the items correctly answered by Lin on the left of the page and those that he answered wrongly on the right. For incorrect items, the wrong option selected by the pupil is indicated in brackets.

Four corners of a kidmap are marked by drawing a dotted line across the left of the page at the top of the region of uncertainty about the pupil's achievement on the test and across the right of the page at the bottom of this region. The four corners defined in this way ensure that items in the top left corner are substantially above the pupil's estimated achievement level and those in the bottom right corner are significantly below his estimated achievement level. The results of most interest will be those at the bottom right of a pupil's report. These will be items that most pupils found easy but that this pupil either did not answer or answered incorrectly. For Lin, the question that appears in the bottom right corner of his kidmap is item 1 of Geometry. He appears to have problem with the fundamental idea of straight lines and curves.

Figure 2. Example of Kidmap for G3 Geometry



▭ indicates the ability estimate of the pupil
Dotted lines are drawn at plus and minus one standard error

Conclusion

This study explored a logistic test model such as the Rasch Model for a basis of criterion-referenced testing based on item banking. There is ample room for increasing, refining and validating the pool of test items that will fit the theoretical model for this study using a much larger sample. Although care had been taken to select the G3 and G5 samples as representative for this project, limitations could arise from the moderate sample sizes of 151 and 139 pupils. A much larger sample would be more appropriate if the objective was to adopt a sample-free approach to the pooling of items to give a greater degree of freedom to assess pupils according to any syllabuses.

The *kidmaps* produced in this study, are helpful for teachers prior to instruction, in order to enhance their understanding of where their pupils' strengths and weaknesses lie in the various aspects of the mathematics curriculum. Due to the difference in mathematical nature of the three aspects of mathematics: number, measurement and space, which all children must experience by the end of the primary school years, a particular child may achieve highly in one aspect and not in another, as shown in this study. The report to parents could provide qualitative information on the achievements of individual students in these key areas of the curriculum. While a single percentage score does not tell parents very much exactly how their children fared in the subject, the report to parents produced in this study aims to be able to give concerned parents a detailed breakdown of the specific concepts, skills or processes that a child had or had not achieved in all the three topics.

References

Adams, R.J. & Khoo, S.T. (1991). *Quest: The interactive test analysis system*. Hawthorn, Victoria: ACER.

Cornish, G. & Wines, R. (1977). *Mathematics profile series. Operations test teachers handbook*. Hawthorn, Victoria: ACER.

Foong, P.Y., Yap, S.F., Khoo, P.S. & Kaur, B. (1994). *Levels of attainment in number, measurement, geometry for pupil performance profile in the primary school*. Unpublished Research Report, National Institute of Education, Singapore.

Kwan, P.Y.K. & Shannon A.G. (1989). Objective tests and Latent Trait theories. *International Journal of Mathematical Education in Science and Technology,* 20, 457-67.

Law, B. (1984). *Uses and abuses of profiling. A handbook on reviewing and recording student experience and achievement.* London: Harper & Row.

Malone, J., Douglas, G., Kissane, B. & Mortlock, R. (1980). Measuring problem-solving ability. In S. Krulik & R. Reys (Eds.), *Problem solving in school mathematics - 1980 yearbook.* Reston: National Council of Teachers of Mathematics.

Masters, G., Lokan, J., Doig, B., Khoo, S.T., Lindsey, J., Robinson, L. & Zammit, S. (1990). *Profiles of learning: The Basic Skills Testing Program in New South Wales, 1989.* Hawthorn, Victoria: ACER.

Masters, G. & Doig, B. (1992). Understanding children's mathematics: Some assessment tools. In G. Leder (Ed.), *Assessment and Learning of Mathematics* (pp. 249-267). Hawthorn, Victoria: ACER.

Stiggins, R.J. (1988). Revitalising classroom assessment: The highest instructional priority. *Phi Delta Kappa, 69*, 363-372.

Willmott, A.S. & Fowles, D.E. (1974). T*he objective interpretation of test performance*. Berkshire: National Foundation for Educational Research.

Appendix A
All Items of the G3 Geometry Test

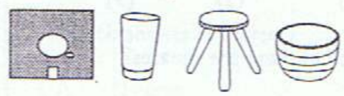
1. Look at these letters.

D E W U

Which of these letters are made up of only straight lines?

(A) E only
(B) D and E only
(C) E and W only
(D) W and U only

2. Look at these objects.

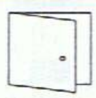


(1) (2) (3) (4)

Which of these have a curved face?

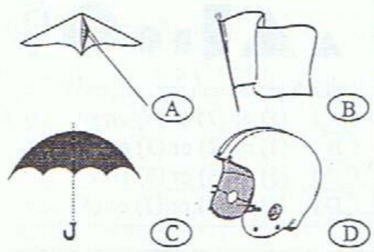
(A) (1) and (3) only
(B) (2) and (4) only
(C) (2), (3) and (4) only
(D) (1), (2) and (4) only

3.

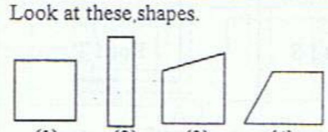


The corner of a door is a right angle.

In which of these objects are there right angles?



4. Look at these shapes.

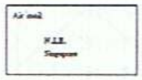


(1) (2) (3) (4)

Which of these shapes have a right angle, and a pair of opposite sides equal?

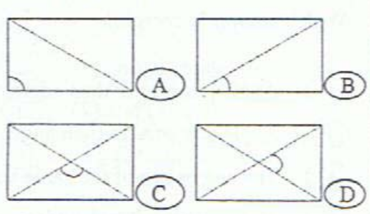
(A) (1) and (2) only
(B) (1) and (3) only
(C) (3) and (4) only
(D) (1), (2) and (3) only

5.

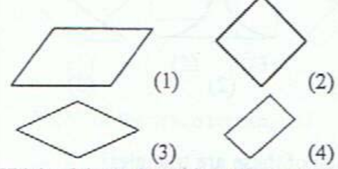


The corner of an envelope is a right angle.

Which of these show an angle bigger than a right angle?



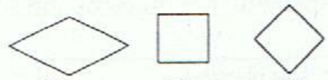
6. Look at these shapes.



Which of these shapes have four equal sides?

(A) (3) only
(B) (2) and (3) only
(C) (2) and (4) only
(D) (1), (2) and (3) only

7. Look at these diagrams.

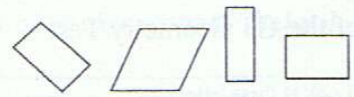


(1) (2) (3)

Which of these are squares?

- (A) (2) only
- (B) (3) only
- (C) (2) and (3) only
- (D) (1), (2) and (3)

8. Look at these diagrams.

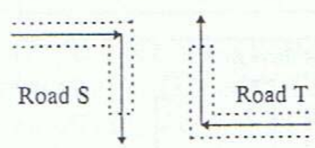


(1) (2) (3) (4)

Which of these are rectangles?

- (A) (1) and (3) only
- (B) (3) and (4) only
- (C) (1), (3) and (4) only
- (D) (1), (2) and (4) only

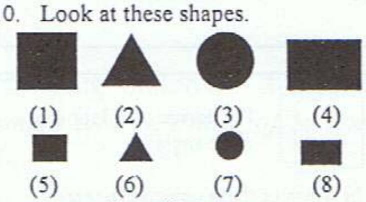
9 There is a bend in each of these roads.



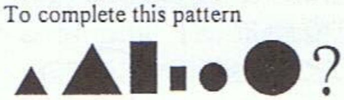
Which answer is correct?

- (A) Road S bends more than road T.
- (B) Road S bends less than road T.
- (C) Roads S and T bend the same.
- (D) You cannot tell by looking.

10. Look at these shapes.



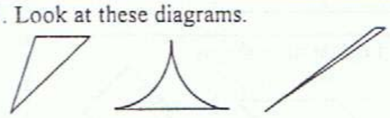
To complete this pattern



which shape will you choose?

- (A) (1) or (2) or (4) only
- (B) (1) or (2) or (3) or (4) only
- (C) (5) or (6) or (8) only
- (D) (5) or (6) or (7) or (8) only

11. Look at these diagrams.

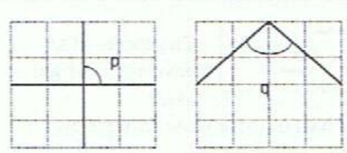


(1) (2) (3)

Which of these are triangles?

- (A) (1) only
- (B) (1) and (2) only
- (C) (1) and (3) only
- (D) (1), (2) and (3)

12. Compare the two angles.



Which answer is correct?

- (A) Angle p is bigger than angle q.
- (B) Angle p is smaller than angle q.
- (C) The angles are of the same size.
- (D) You cannot tell by looking.

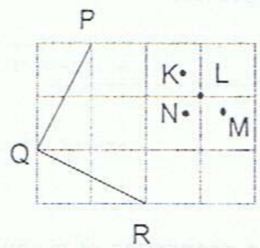
13. Which point K, L, M or N must be joined to P and R so as to draw a square?

(A) K

(B) L

(C) M

(D) N



14. Read these statements.

A triangle will
(1) always have three sides.
(2) sometimes have three equal sides.
(3) sometimes have two equal sides.

Which answer is correct?

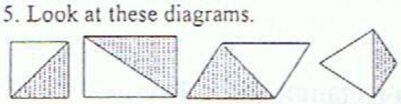
(A) (1) only

(B) (1) and (2) only

(C) (1) and (3) only

(D) (1), (2) and (3)

15. Look at these diagrams.



(1) (2) (3) (4)

Which of these contain a triangle with two equal sides?

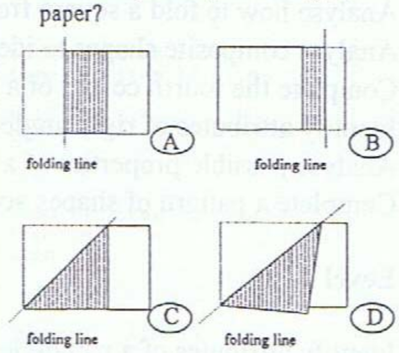
(A) (1) and (3) only

(B) (1) and (4) only

(C) (1), (2) and (3) only

(D) (1), (3) and (4) only

16. A square can be *folded* from a piece of paper. Which diagram below shows the *first step* in folding the paper?



17. Read these statements.

A rectangle will
(1) always have four right angles.
(2) always have four equal sides.
(3) sometimes have four equal sides.

Which answer is correct?

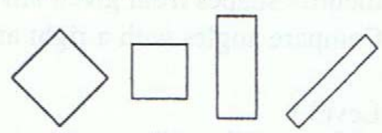
(A) (1) only

(B) (1) and (2) only

(C) (1) and (3) only

(D) (1), (2) and (3)

18. Look at these shapes.



(1) (2) (3) (4)

Which of these are rectangles?

(A) (3) only

(B) (3) and (4) only

(C) (2), (3) and (4) only

(D) (1), (2), (3) and (4)

Appendix B

G3 - Geometry Skill Levels and Item Properties

↑ High Geometry Skills

Level 4

- Abstract the relationship between the properties of a square and a rectange (items 17 & 18)

Level 3

- Analyse how to fold a square from a rectangle (item 16)
- Analyse composite shapes to identify a particular shape (item 15)
- Complete the fourth corner of a square on a grid (item 13)
- Identify attributes of right angles drawn on square grids (item 12)
- Analyse possible properties of a triangle (items 11 & 14)
- Complete a pattern of shapes according to one or two attributes (item 10)

Level 2

- Identify attributes of a rectangle (item 8)
- Identify attributes of a square (item 7)
- Identify shapes from given attributes (item 6)
- Compare angles with a right angle or with each other (items 5 & 9)

Level 1

- Recognise a right angle and opposite equal sides within a figure (item 4)
- Recognise a right angle in familiar objects (item 3)
- Identify objects which have a curved face (item 2)
- Distinguish between a straight line and a curved line (item 1)

↓ Low Geometry Skills

Appendix C

G3-GEOMETRY: Scale of pupil achievement and item difficulties

```
------------------------------------------------------------------------------
Item Estimates (Thresholds)        All on all (N = 151, L = 18)
------------------------------------------------------------------------------
                                        ^
  3.0     higher achievement            |        more difficult items
                                        |
                                        |item 18
                                        |
                                        |
  2.0                                   |
                                        |
                                        |item 17
                                       X|
                                        |
                                        |
  1.0                                   |
                                  XXXXXX|
                                        |
                                        |item 15  item 16
                                   XXXXX|item 12  item 13  item 14
                                        |
                               XXXXXXXXX|item 11
                                        |item 10
   .0                        XXXXXXXXXXX|
                                        |item 9
                       XXXXXXXXXXXXXXXXX|
                                        |item 7   item 8
                        XXXXXXXXXXXXXXXX|item 6
                                        |item 5
                XXXXXXXXXXXXXXXXXXXXXXXX|
 -1.0                                   |item 4
                                        |
        XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX|item 2   item 3
                                        |
                            XXXXXXXXXXXX|item 1
                                        |
                                 XXXXXXX|
 -2.0                                   |
                                        |
                                     XXX|
                                        |
                                        |
 -3.0                                   |
                                        |
                                       X|
                                        |
                                        |
          lower achievement             |        less difficult items
------------------------------------------------------------------------------
  Each X represents    1 student
```